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To whom it may concern:

I have reviewed the Covered Assets strategy of the Wealth Building Cornerstones™ system and conclude that it is consistent with the strategy named “Scenario 3: Investments + Single-Life SPIA + Whole Life” in the paper I wrote titled “Optimizing Retirement Income by Combining Actuarial Science and Investments” in May 2015.

Sincerely

Wade Pfau

Professor of Retirement Income, The American College

270 S Bryn Mawr Ave.

Bryn Mawr, PA 19010

610-526-1569

Optimizing Retirement Income by Combining Actuarial Science and Investments

by

Wade D. Pfau, Ph.D., CFA

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Author Bio

Wade D. Pfau, Ph.D., CFA, is a Professor of Retirement Income at The American College for Financial Services in Bryn Mawr, PA. He also serves as the Director of Retirement Research for McLean Asset Management and in Stream Solutions. He is a past selectee for the *InvestmentNews* “Power 20” in 2013 and “40 Under 40” in 2014, the *Investment Advisor* 25 list for 2014, and *Financial Planning* magazine’s Influencer Awards. He is a two-time winner of the *Journal of Financial Planning* Montgomery-Warschauer Editor’s Award, a two-time winner of the Academic Thought Leadership Award from the Retirement Income Industry Association, and a best paper award winner in the Retirement category from the Academy of Financial Services. He is also a contributor to the curriculum of the Retirement Income Certified Professional (RICP) designation. He is a co-editor of the *Journal of Personal Finance*. He has spoken at the national conferences of organizations such as the CFA Institute, FPA, NAPFA, AICPA-PFP, and AFS. He holds a doctorate in economics from Princeton University and publishes frequently in a wide variety of academic and practitioner research journals. He hosts the [Retirement Researcher](#) website, and is a monthly columnist for *Advisor Perspectives*, a RetireMentor for MarketWatch, a contributor to *Forbes*, and an Expert Panelist for the *Wall Street Journal*. His research has been discussed in outlets including the print editions of *The Economist*, *New York Times*, *Wall Street Journal*, and *Money Magazine*.

Executive Summary

We investigate whether more efficient retirement income solutions can be obtained through careful efforts to combine investment portfolios, income annuities, and whole life insurance into an overall retirement income plan. A basic investment portfolio allocates assets between stocks and bonds. Stocks are volatile investments which focus on growth, and bonds are generally viewed as a way to diversify and reduce overall portfolio volatility. The benefits from investment strategies are liquidity and upside growth potential. But investments alone do not necessarily create an efficient retirement plan. By efficiency, we mean that there may be an alternative way to structure retirement assets and life insurance during working years, so as to be able to support a higher level of retirement spending as well as an equal or greater amount of financial assets to be available as part of a legacy.

Actuarial science principles can contribute to better retirement outcomes. Actuarial science allows personal retirement planning to be treated more like a defined-benefit pension plan. These plans can pool financial market risks between different cohorts and can pool longevity risk between different individuals within the same cohort. By including actuarial science, longevity-protected spending can be determined in advance through these pooling mechanisms. In contrast, those relying on their own devices to manage market and longevity risks must behave conservatively with regard to market return assumptions and the planning horizon, lest they run out of assets. And even with conservative spending assumptions, investment portfolios do not have guarantees and remain vulnerable to depletion.

To compare with investments, we can think of the combination of whole life insurance and income annuities as “actuarial bonds” with an average maturity equal to life expectancy. These financial products, which invest primarily in a fixed income portfolio, can better hedge a

retiree's personal income needs. By combining them, the overall planning horizon can essentially be fixed at something close to life expectancy, as whole life insurance provides a higher implied return when the realized lifetime is short, and income annuities provide a higher return when the realized lifetime is long. This is a more effective way to use fixed income assets than as a portfolio volatility reduction tool.

We confirm these statements through case studies with 35-year old and 50-year old couples, comparing three retirement scenarios for each couple. The **first scenario** uses a term life policy to meet life insurance needs until retirement, and otherwise draws retirement income with systematic withdrawals from an investment portfolio. The **second scenario** also uses term life insurance. However, this scenario adds partial annuitization with a joint-life income annuity to provide income along with portfolio withdrawals from the remaining non-annuitized assets. The **third scenario** maintains a permanent death benefit with whole life insurance, and uses a single-life income annuity along with systematic withdrawals from the remaining non-annuitized assets for retirement income. Retirees may feel more comfortable with the idea of partial annuitization when their household balance sheet also includes whole life insurance in retirement, because the death benefit from the whole life insurance may be viewed as a “refund” of the monies used for the income annuity.

By tracking the course of income and legacy wealth through age 100 for each scenario, we find that the inclusion of an income annuity can allow for greater income throughout retirement. Our simulations show that the risk pooling features of the income annuity are essentially a more significant factor in boosting retirement income than is the greater upside potential offered through increased reliance on investments. This is because incorporating the whole life insurance, even though it requires larger premiums than the term life insurance,

supports a higher income level (by justifying partial annuitization) while also supporting a larger legacy. Traditionally there is a tradeoff between enjoying more income and leaving a larger legacy, but this integrated approach actually allows for increases in both income and legacy. We can indeed conclude that an integrated approach is a more efficient retirement income strategy.

We find substantive evidence that an integrated approach with investments, whole life insurance, and income annuities can provide more efficient retirement outcomes than relying on investments alone. Because whole life insurance can play an important role in producing more efficient retirement outcomes, younger individuals planning for both retirement and life insurance needs may view whole life insurance in a new light as a powerful retirement income planning tool. The recent conventional wisdom of “buy term and invest the difference” is less effective than many realize when viewed in terms of the risk management needs of a retirement income plan.

Introduction

Retirement income planning has now emerged as a distinct field in the financial services profession, though it is still in its infant stage and undergoing growing pains. Though not yet recognized by all players, one matter is very clear, and it is that the financial circumstances facing retirees differ dramatically from pre-retirees. For instance, retirees face reduced flexibility to earn income in the labor markets as a way to cushion their standard of living from the impact of poor market returns. Retirees are also seeking specifically to create an income stream from their assets, and this is an important constraint on their investment decisions which was not at the forefront for those still working and accumulating assets. Retirees experience heightened vulnerability to sequence of returns risk once they are spending from their investment portfolio: poor returns early in retirement mean that the sustainable withdrawal rate from a portfolio may fall well below what is implied by average portfolio returns over the whole retirement period. One simply cannot approach retirement income planning by using a linear or constant return assumption in a spreadsheet. Though if one does insist on using a constant return assumption to make a retirement plan in a spreadsheet, Pfau (2014) provides guidance on the choice by quantifying how it is important to substantially reduce the return assumption below the expected average, especially in retirement. This is a point which many investment management professionals have not internalized into their thinking, as they are conditioned to using their idea about average returns as the input. These issues suggest that retirees have reduced risk capacity relative to pre-retirees. Their standard of living is more vulnerable to market volatility and extra caution is warranted.

Another pivotal matter confronting retirees is that the length of one's retirement is unknown and it could be much shorter or much longer than a person's statistical life expectancy.

Retirees need a plan to manage this longevity risk. Inflation is another concern. Even low inflation will compound to have a big impact over a lengthy retirement, leaving retirees vulnerable. Retirees must also preserve flexibility and liquidity to manage risks related to unplanned expenses. Finally, a retirement income plan should build in provisions for the unfortunate reality that many will experience declining cognitive abilities at advanced ages, which could hamper portfolio management and other financial decision-making skills.

Because retirement income planning is still a relatively new field, there remains a rift in the financial services profession about the best approach for building a retirement income plan. Pfau and Cooper (2014) describe two fundamentally different philosophies toward retirement income planning, which they categorize as probability-based and safety-first. Though not discussed along these lines within their paper, a noticeable distinction between these philosophies involves where an individual decides to place their trust: the risk/reward possibilities of an equity portfolio, or the contractual guarantee of insurance products.

Those favoring investments rely on the notion that the market will eventually provide favorable returns for most every retiree. Though stock markets are volatile, over a reasonable amount of time, stocks can be expected to outperform bonds. Those believing strongly in investments see upside potential from an investment portfolio as so significant that there is a very limited role for insurance solutions in a retirement income plan. Why needlessly cut off the upside? On the investment side, there is also a general unease about relying on the long-term prospects of insurance companies or bond issuers to meet their contractual obligations. Perhaps not fully understanding the implications for how sequence risk differs from market risk (more on this later), the belief is that in the rare event the performance for the equity portfolio does not

materialize, it would imply an economic catastrophe that would sink insurance companies as well.

Meanwhile, those favoring insurance believe that contractual guarantees are reliable, and that an overreliance on the assumption that favorable market returns will come can be emotionally overwhelming as well as dangerous for retirees. Indeed, there is greater concern about the implications of market risk. Even if there is a “low” probability of portfolio depletion, each retiree only gets one opportunity for a successful retirement. At the very least, essential income needs should not be subject to the whims of the market. As well, advocates from this side view investment solutions as undesirable because the retiree retains all the longevity risk. This is a risk which an insurance company is in a better position to manage and handle.

For retirement income, we must step away from the notion that either investments or insurance alone will best serve retirees. More emphasis is needed on the basic forms of insurance products, and how they may behave as part of an integrated retirement income plan. We extend research on the efficient frontier for retirement income which has been explored by Chen and Milevsky (2003), Pfau (2013), and others, by also including life insurance alongside income annuities and investments. While there are a range of complex insurance solutions available, as well as investment-only solutions, we consider combining financial assets, income annuities, and whole life insurance. None of these basic insurance and investment products can offer comprehensive solutions on their own, but they fit well together into an overall strategy for retirement.

Traditional bonds do not necessarily make sense for retirement portfolios. However, we view the combination of whole life insurance and income annuities as an “actuarial bond” with an average maturity equal to life expectancy. Then, instead of thinking in terms of stocks and

bonds, we focus on stocks and “actuarial bonds,” simplifying retirement planning by calibrating the planning horizon **to life expectancy**. Tracking the course of income and legacy wealth through age 100, we find that the inclusion of an income annuity can allow for more income throughout retirement. The risk pooling features of the income annuity are essentially of greater importance than the greater upside potential available with more reliance on investments. As well, the whole-life policy, though requiring greater premiums than a term-life policy, supports a higher income level throughout retirement, as well as a larger legacy. Indeed, we find substantive evidence that an integrated approach with investments, whole life insurance, and income annuities can provide more efficient retirement outcomes than relying on only investments.

The Retirement Income Challenge

A retirement income strategy seeks to meet the specific financial goals identified by a retiree in a way which can best manage the wide variety of risks which threaten those goals. The primary financial goal for most retirees will relate to their spending: maximize spending power [**lifestyle**] in such a way that spending can remain consistent and sustainable without any drastic reductions, no matter how long the retirement lasts [**longevity**]. Other important goals may include leaving assets for subsequent generations [**legacy**], and maintaining sufficient reserves for unexpected contingencies [**liquidity**]. With those bracketed terms, financial goals for retirement can be summarized as (1) lifestyle, (2) longevity, (3) legacy, and (4) liquidity. The risks confronting these goals are numerous. Key risks most relevant to our discussion include:

Longevity Risk: The fundamental risk for retirement is longevity. How long must a retirement plan be able to provide income? That is, how long will the retiree live? A long life is wonderful, but it is also more costly and a bigger drain on a retiree’s resources. We can identify our

statistical life expectancies, which is how long we can expect to live on average. But half of the population will live past their life expectancy, and some individuals will live substantially longer.

With investments, this risk can only be self-managed through a conservative income plan based on spreading assets over a longer period than life expectancy. The alternative is to transfer this risk to an insurance company, so that those with short lives subsidize those with long lives, a possibility which can be attractive when an individual doesn't know in advance to which group they will fall. This risk transfer lets groups of people, those living shorter than life expectancy and those living longer than life expectancy, benefit from the higher income rate while alive regardless of how long each lives. These higher income rates, or subsidies, are possible due to the "mortality credits" or subsidies described above.

Investment Volatility and Sequence-of>Returns Risk: The financial market returns experienced near one's retirement date matter a great deal more than most people realize. Even with the same average returns over a long period of time, retiring at the start of a bear market is very dangerous because wealth can be depleted quite rapidly as withdrawals are made from a diminishing portfolio and little may be left to benefit from any subsequent market recovery. We do not have the luxury of forecasting whether there will be a bear or bull market at the onset of our retirement.

Sequence of returns risk relates to the heightened vulnerability individuals face regarding the realized investment portfolio returns in the years around their retirement date. Though this risk is related to general investment risk and market volatility, sequence of returns risk differs from general investment risk. The average market return over a 30-year period could be quite generous, but if negative returns are experienced in the early stages when someone has started to

spend from their portfolio, sequence of returns risk manifests through the fact that the early portfolio decline creates a subsequent hurdle that cannot be overcome even if the market is offering higher returns later in retirement. The dynamics of sequence risk suggest that the retirement prospects for a particular cohort of retirees could be jeopardized by a prolonged recessionary environment early in retirement without there necessarily being an accompanying economic catastrophe. This is a subtle but important point worth repeating. Particular retiree cohorts could experience very poor retirement outcomes relative to those retiring a few years earlier or later, and devastation for one cohort does not necessarily imply devastation for an insurance company which is pooling financial market risk across different cohorts.

Inflation Risk: Retirees face the risk that inflation will erode the purchasing power of their savings as they progress through retirement. Even with just 3% average annual inflation, the purchasing power of a dollar will fall by more than half after 25 years.

Unexpected Spending Risk: When attempting to budget over a long retirement period, it is important to include allowances for contingencies. Unexpected expenses could relate to any number of matters, including health and long-term care needs, fraud and theft, a need to help other family members, changes in public policy, divorce, changing housing needs, home repairs, and rising prescription costs.

Cognitive Decline Risk: Finally, an important matter is the reality that many retirees will experience cognitive decline, which will compound the difficulties associated with long-term financial planning. Finke, Howe, and Huston (2011) prepared a study of households over age 60. They found that basic financial literacy skills decline by about 2% per year after age 60, and that this can be attributed to cognitive changes. At the same time, confidence about one's abilities to make prudent financial decisions does not decline with age. It will be increasingly difficult for

individuals to make good portfolio investment and withdrawal decisions as they enter into advanced ages.

Related to this risk is the reality that many households do not share equally in the responsibilities regarding the management of personal finances. When the spouse who is primarily responsible for managing the finances dies first, it reflects a real problem for the surviving spouse if a clear plan is not in place. The surviving spouse will be vulnerable to making financial mistakes and to falling victim to various financial predators. An income annuity, or clearly written instructions on how to purchase an income annuity, should provide better protection for an unsophisticated spouse who may be overwhelmed by the complex problem of managing investments and withdrawals.

After determining a person's retirement objectives and identifying key risks which need to be managed, the next step is to identify the tools for crafting the retirement income strategy. Most Americans will have access to at least some of the following tools: Social Security benefits, Medicare, company pensions, a social support network, and part-time work. For those desiring more income in retirement than they can obtain through these sources, it is necessary to save for retirement as well. Savings can be allocated between investment and insurance tools. On the investment side, clients can build a diversified portfolio of stocks, bonds, and alternative investments. Once retired, retirees could use a systematic withdrawal strategy to spend from their investment portfolio. Retirees may also utilize income methods that include the integration of an investment portfolio with income annuities and whole life insurance. Let's explore the advantages and disadvantages of various retirement income methods.

The Advantages and Disadvantages of Investments in Retirement

Because managing diverse risks in retirement requires an integrated approach, building a retirement income strategy with only investments or with only insurance can be rather inefficient. Each approach has its advantages and disadvantages. First, with investment solutions, a higher lifestyle may be supported if one is willing to spend and invest aggressively in the hope of subsequently earning higher market returns to support a higher income rate. And should decent market returns materialize, investment solutions can be sustained indefinitely. Portfolio balances are also liquid in the sense that they are accessible to the retiree and are not locked away as part of a contractual agreement. The ability to generate upside growth with investments could also support a larger legacy.

However, the dual impact of sequence and longevity risk creates a very real possibility with investments that one cannot support their desired lifestyle over the full retirement period. These are risks which a retiree cannot be compensated for taking. The risks can be easily pooled. Investment approaches seek to manage sequence and longevity risk by having the retiree spend conservatively. Retirees spend less as a way to avoid depleting their portfolio through a bad sequence of returns in early retirement, and they also spend less because they must plan to live well beyond their life expectancy.

Longevity protection is not guaranteed with investments and sufficient assets may not be available to support a long life or legacy. This is particularly important in light of the ongoing improvements in mortality, which means that today's retirees will live longer than those from earlier cohorts. As well, individuals experience declining cognitive abilities with age, which can make it more difficult to manage an investment portfolio. Assets may also be less liquid than

they appear. Though they are technically liquid, a retiree who spends assets that were meant to cover spending needs later in life may find that those later needs can no longer be met.

The Advantages and Disadvantages of Insurance in Retirement

Insurance companies can pool sequence and longevity risks across a large base of retirees, allowing for retirement income spending that is greater than both the sustainable withdrawal rate and average long-term fixed income returns. This may support a higher lifestyle than what is feasible for someone self-managing these risks. Guarantees can also provide a peace of mind which leads to a less stressful and more enjoyable retirement experience. Overly conservative retirees become so concerned with running out of money that they spend significantly less than they could, and a monthly annuity payment can provide the permission to spend and to enjoy retirement.

Longevity protection is a primary benefit from an insurance solution, as it provides a guaranteed income for as long as one lives. It hedges longevity risk and calibrates the planning horizon to something much closer to life expectancy. Those who do not live long lives subsidize the income payments to those who do live longer than life expectancy. However, as stated previously, both groups enjoy higher income rates because they have shared their risk of longevity. This higher income also provides flexibility to spend less than possible and maintain more reserves to manage inflation risk. As for liquidity, the cash value of the whole life policy is a liquid asset which can be spent without adding to taxable income (like a Roth IRA). However, utilization of the cash value of the policy via a policy loan does reduce the death benefit. The cash value could be used as a contingency fund. A death benefit is also created with life insurance to provide a specific legacy amount. An income annuity dedicates assets specifically

toward the provision of income, allowing other assets to be earmarked specifically for growth. This can allow for a larger legacy than otherwise, especially when the retiree enjoys a long life.

On the other hand, though income annuities can guarantee a lifestyle, they lack the ability on their own to provide upside potential, and inflation-protected versions can be costly in current markets. More aggressive retirees may simply conclude that the loss of upside potential is too great a sacrifice. Income annuities and life insurance death benefits generally are not liquid assets. Though the death benefit may grow over time, it is not likely to keep pace with inflation. Finally, income annuities do not offer legacy benefits without adding additional riders which reduce the power of mortality credits.

With their varying advantages and disadvantages, we now analyze strategies which combine insurance and investments.

Methodology and Assumptions for the Case Study: 35-Year Olds Steve and Susie

Steve and Susie are a married 35-year-old couple with two children. They have determined that it is time to get serious about retirement and life insurance planning. Steve is employed and Susie is a homemaker. Steve is seeking an additional amount of life insurance death benefit of about \$400,000 that, along with his other life insurance, will support his family in the event of his death prior to age 65.

Steve presently has \$65,000 saved in a 401(k) plan with his employer, which is invested with an equity glide path strategy matching a typical target date fund. He would like to plan for retirement at 65, and he believes it will be possible to set aside \$15,000 per year from his salary (future savings will be adjusted for inflation as his salary grows) over the next 30 years until his

planned retirement date. Steve expects to be in the 25% marginal tax bracket in both his pre-retirement and post-retirement years.

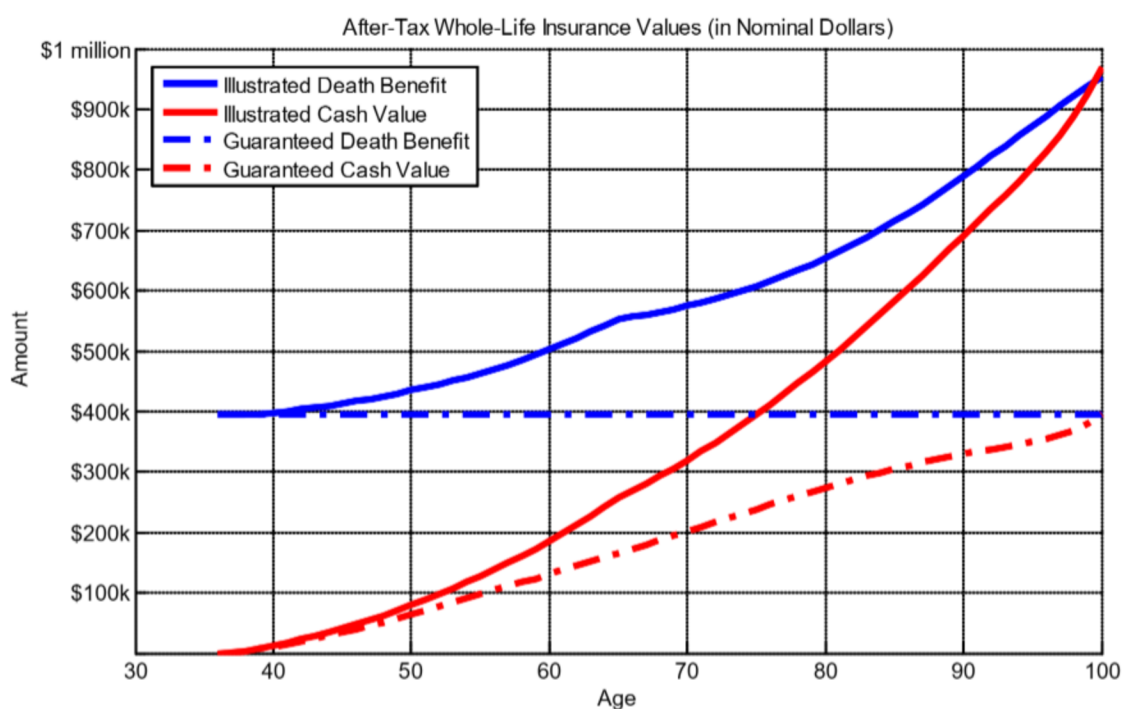
Steve must decide whether to purchase a term life insurance policy to increase his existing coverage to meet his economic capital replacement value for his family, or to otherwise purchase a whole life insurance policy which can serve his additional economic capital replacement value, as well as being integrated into his retirement income strategy. From the \$15,000 he can set aside for his insurance and retirement planning needs, he will pay for life insurance premiums and the taxes to cover those premiums (at a 25% marginal tax rate), and the remainder will go into his tax-deferred 401(k).

In all scenarios, we assume that Steve is directing at least enough to the 401(k) to satisfy the conditions for the highest possible company match, though we do not specifically model any company match when simulating retirement income. An employer match would increase income proportionately for all our scenarios. More generally, Steve and Susie may also have other resources in retirement which we are not analyzing. We are modeling all of the relevant features about how to best make the investment and insurance decisions for the \$15,000 annual set-aside to meet life insurance needs and to obtain the most desirable retirement outcomes from this portion of their household resources.

For life insurance, Steve will decide between term life and whole life insurance. The term life policy he considers is a 30-year policy with a \$394,737 death benefit with an annual premium of \$539. Taxes on the pre-tax income required to cover this premium are \$180. After paying the term life premium and taxes, he would contribute the remaining \$14,281 per year to his 401(k).

The whole-life policy Steve considers also carries an initial death benefit of \$394,737 and the whole life insurance annual premium is \$4,500. The non-guaranteed provisions of the policy contain the potential for a premium offset which Steve could use, and so he would pay premiums for 30 years to age 65, and then could use part of the policy's non-guaranteed dividends to pay the subsequent premiums through age 100. Assumptions for the whole-life policy are based on an illustration prepared by OneAmerica in March 2015, assuming the natural premium offset starting at age 65. The nominal values for the death benefit and cash value (both illustrated and guaranteed) are shown in Figure 1. Taxes to cover the whole-life premium are \$1,500, and so with a whole-life policy Steve can contribute \$9,000 per year to his 401(k) after setting aside \$6,000 for whole life insurance premiums and taxes.

Figure 1
After-Tax Whole Life Insurance Values



Upon reaching age 65 in 30 years, Steve and Susie will consider whether a single-premium immediate annuity (SPIA) might be a worthwhile addition to their retirement income plan. Income annuities offer a variety of options regarding whether income starts immediately or is deferred, whether income covers a single life or joint lives, whether there is a certain payment for a set number of years, whether any cost-of-living adjustments will be made to income, and whether cash or installment provisions are included in the event of an early death. To simplify our analysis, we consider two basic possibilities: Steve buys a single life-only immediate annuity at 65 on his life, or Steve and Sue buy a joint life and 100% survivor annuity on them both. Neither SPIA contract will have a cost-of-living adjustment, and in both cases the annuities would be purchased with qualified funds after Steve has stopped working and completes a rollover from his 401(k) to a traditional IRA.

The single life-only versions for SPIAs offer the highest payout rates (income) because they provide the most “mortality credits” (those dying earlier provide a source of money to those who live longer, justifying a higher payout rate). It is difficult to predict annuity rates in 30 years. Those rates will depend on interest rates and mortality projections at that time. Our market return simulations, which are described in the appendix, do allow for interest rates to increase on average from their currently low levels, suggesting on average that SPIA rates will be higher in 30 years than they are today. On the other hand, longevity improvements over the next 30 years will likely create downward pressures on annuity rates separate from any interest rate changes. On January 20, 2015, the Cannex annuity quotation service identifies that the average of the top three quotes for a 65-year old male single-life only annuity is a 6.74% payout rate. For a joint-life annuity, the average for the top three quotes is a 5.6% payout rate. On the same date, 10-year Treasury yields were only 1.8%, compared to their historical average (see appendix) of 4.7%.

Though it requires an element of guesswork for the reasons described before, we conclude it is reasonable to add 1% to the payout rates relative to today's quotations in order to estimate the payout rate in 30 years. We assume that the SPIA payout rates in 30 years are 7.74% for the male single-life only version, and 6.6% for the joint-life & 100% survivor version. The single-life only SPIA provides 17% more income for a given premium relative to the joint-life SPIA.

As for investments, the Sustainable Withdrawal Rate Method uses 50,000 Monte Carlo simulations for investment returns based on today's lower interest rates, with built-in provisions to allow interest rates and market returns to trend back toward their higher historical averages over time. The methodology to create these simulations is more technical in nature and is provided in the paper's appendix.

With the asset return projections, Steve invests his 401(k) assets in a target date fund with asset allocation and expense ratios matching the averages found by Morningstar (2014) in their survey of target-date fund families. These are the averages across the available target-date fund families in 2013, with age 65 (2045) calibrated as Steve's target date. These calculations lead to a stock allocation of 89% at age 35, and the stock allocation gradually decreases to 48% by age 65. The stock allocation continues to decrease after retirement, leveling off at 40% by age 73. As target-date funds exist only in five-year increments, we make a linear interpolation for asset allocations in between each five-year fund date. We also set mutual fund fees equal to the 0.84% average portfolio administration cost as determined by Morningstar in the same report. A financial advisory fee of 0.75% is also charged to these assets. Therefore, the total fees equal 1.59% on all retirement savings assets.

A review of the tax principles used herein is also in order. Investments are made in Steve's tax-deferred 401(k) plan. This means that taxes are not paid initially on the plan

contributions, but any withdrawals from the plan will be subject to ordinary income tax rates. At retirement, Steve completes a rollover of his 401(k) to a traditional IRA. This is not a taxable event. With a tax deferred account, the government effectively owns a portion of the account as identified by the tax rate. Taxes are deferred until withdrawals are made. The legacy value of the IRA is in pre-tax terms. Therefore, the after-tax value of the IRA would have to consider ordinary income tax ramifications to determine the actual net after-tax value of these monies.

Life insurance premiums are paid with post-tax funds. But no taxes are due on the death benefit, making it a post-tax number. As well, a life insurance policy can be arranged so that funds can be borrowed from the cash value without being taxed, which as described earlier, does impact the death benefit. It should be noted that another use of life insurance within a retirement income strategy is sourcing the income from the policy's cash value in years after market downturns, in order to avoid selling financial assets at depressed prices. For this research, we will not use the cash value in such a way, and so it is not necessary to further expound upon this tax feature.

If an immediate income annuity is purchased at retirement, this purchase is made with qualified funds inside the traditional IRA. Annuity income is then fully taxable at income tax rates as it leaves the qualified account. Because the annuity is purchased in a qualified account, someone seeking to purchase an annuity with funds equivalent to the life insurance death benefit would actually need to inflate their purchase to account for the differing tax treatment. For example, a non-taxed death benefit of \$100,000 is equivalent to $\$100,000 / (1 - \text{tax rate})$ in a qualified account. With a tax rate of 25%, this is \$133,000. Annuitizing this larger amount would then be equivalent to the amount of the death benefit on a post-tax basis.

We are now ready to consider three scenarios for Steve and Susie as follows.

Scenario 1: Investments and Term-Life Insurance

The first scenario is the typical “buy term and invest the difference” case. Term insurance is used for economic capital protection, and its smaller premium allows for a greater amount to be contributed to the tax-deferred account. Financial assets are invested in a target date fund as described before. The term policy expires at Steve’s planned retirement age of 65.

With the accumulated investment assets, retirement income will be generated with a systematic withdrawal strategy. Steve seeks annual spending adjustments which match the Consumer Price Index. When it comes to building a retirement income strategy with investments, the starting point is William Bengen's 4% rule. Bengen (1994) initiated a line of research in which he found that an investor with 50-75% stocks who does not pay any investment management fees could sustain 30 years of inflation-adjusted spending at a level calibrated to 4% of the initial retirement date account balance. This finding is based on the worst-case from US history when simulating retirements for hypothetical individuals using all the available rolling 30-year periods. Such systematic withdrawal strategies focus on a total returns investment portfolio perspective.

The limited US historical record is not necessarily sufficient for determining a worst-case spending rate from a volatile portfolio, nor is it wise to assume no investment fees will be paid. Pfau (2015) uses Monte Carlo simulations calibrated to lower current interest rates and with a 0.5% annual investment management fee to determine that a 2.88% withdrawal rate has a 10% chance for failure over a 30-year horizon for someone retiring today with a portfolio of 50% stocks and 50% bonds.

Because most investors must pay investment management fees and will not earn the precise underlying indexed market returns, and because interest rates have rarely been as low in

the past as they are today, Bengen's historical simulations do not fully reflect the risks associated with the strategy. But Steve and Susie will not retire for 30 years, and our simulations suggest there is a good chance that interest rates will be higher by the time they retire. As such, we use 3.5% as the "sustainable withdrawal rate" for the systematic withdrawal strategy. Steve's systematic withdrawal strategy is to withdraw 3.5% of his accumulated retirement date assets, and to then take withdrawals in subsequent years which reflect this initial level plus cumulative inflation, for as long as sufficient assets remain. Spending drops to \$0 in the event that the portfolio depletes.

Scenario 2: Investments, Joint-Life and 100% Survivor Income Annuity, Term-Life Insurance

Scenario 2 shares many similarities with Scenario 1. Steve uses the same term policy and invests the remainder in a tax-deferred account, leading to the same retirement date wealth accumulations. The difference happens at the retirement date. In Scenario 2, Steve purchases a joint-life and 100% survivorship income annuity with a premium amount equal to the pre-tax equivalent of the illustrated death benefit for the whole-life policy (\$553,504 in nominal terms, see Figure 1) on an after-tax basis. With a 25% tax rate, the pre-tax amount to annuitize is \$738,005. (Note: Table 2 reflects the proper use of the \$738,005 pre-tax equivalent value) Though Steve did not use the whole-life policy in this scenario, annuitizing this pre-tax equivalent amount allows for a proper comparison with Scenario 3. If Steve's accumulated pre-tax wealth is less than the pre-tax equivalent of his death benefit, then he annuitizes all of his financial assets. Otherwise, he annuitizes this amount and the remaining wealth will be utilized in retirement with the same 3.5% inflation-adjusted systematic withdrawal strategy as described in Scenario 1. The joint-life and 100% survivor income annuity provides income for as long as one member of the couple is alive, and any systematic withdrawals will supplement this income

for as long as financial assets remain. Portfolio depletion is less drastic in this case, as annuity income continues.

Scenario 3: Investments, Single-Life Income Annuity, and Whole-Life Insurance

Scenario 3 contains important differences. First, Steve uses a whole life insurance policy rather than term life insurance policy. Because of the higher premium, he invests less (\$9,000 instead of \$14,281) in his 401(k) plan. Second, Steve buys a single life-only SPIA at 65 with assets from his 401k. He can opt for single-life instead of joint-life, because the death benefit from his whole life insurance will replace those assets upon his death. If necessary, Susie could then use part of the death benefit to buy another single-life income annuity. The difference in annuity payout rates (7.74% instead of 6.6%) allows for 17% more income to be generated by the same annuity premium as compared to Scenario 2. As with Scenario 2, Steve uses accumulated 401(k) assets on an after-tax basis up to the amount of the projected age 65 death benefit to purchase the income annuity. Again, any remaining assets will be utilized with the same 3.5% systematic withdrawal strategy as in the previous scenarios.

A final important difference in Scenario 3 relates to asset allocation. With a whole life policy, the cash value is a liquid asset contained outside the financial portfolio. Essentially, it behaves like fixed income, though it is not exposed to interest rate risk (i.e. the accessible cash value does not decline when interest rates rise). Though it is not precisely the same as holding bonds in an investment portfolio, as there is not a practical way to rebalance the portfolio between stocks and policy cash value, Steve will incorporate the cash value into his asset allocation decisions in order to maintain the target date fund's overall proportion between stocks and "bonds" on a household balance sheet basis. For example, if the target date fund calls for a 60% stock allocation, then the actual stock allocation Steve uses will be 60% times the sum of

the financial portfolio balance and the life insurance cash value, divided by the portfolio balance. Though this could conceivably call for a stock allocation of greater than 100% when the cash value is large relative to the financial portfolio, we constrain the maximum possible stock allocation for the financial portfolio to not exceed 100%.

This is important, because otherwise a strategy which combines an investment portfolio with the same asset allocation as before, with a conservatively invested whole life insurance policy, would create a more conservative overall asset allocation from the household balance sheet perspective. This would reduce the growth potential of the overall strategy. With these adjustments, we are essentially asking whether the fixed income component for household assets should be allocated to only traditional bonds or also to a whole life insurance policy.

Results for the 35-Year Old Couple

Table 2
Outcomes for the Three Scenarios -- 35 Year Old Case

Current 401(k) Savings at Age 35 \$65,000
Amount Set Aside for Savings and Insurance \$15,000

	Scenario 1 Investments No SPIA + Term Life	Scenario 2 Investments + Joint-Life SPIA + Term Life	Scenario 3 Investments + Single-Life SPIA + Whole Life
Term-Life Premiums	\$539	\$539	\$0
Whole-Life Premiums	\$0	\$0	\$4,500
Taxes Paid	\$180	\$180	\$1,500
Remaining Contribution to 401(k)	<u>\$14,281</u>	<u>\$14,281</u>	<u>\$9,000</u>
Totals	\$15,000	\$15,000	\$15,000

All Subsequent Values are Provided on a Pre-Tax Basis (Assuming a 25% tax rate)

Distribution of 401(k) Assets at Age 65			% change from Scenario 1		% change from Scenario 1
10th Percentile	\$918,029	\$918,029	0%	\$741,274	-19%
Median	\$1,673,034	\$1,673,034	0%	\$1,449,785	-13%
90th Percentile	\$3,314,872	\$3,314,872	0%	\$2,950,160	-11%

Illustrated Life Insurance Values at Age 65

Cash Value	\$0	\$0	\$343,116
Death Benefit	\$0	\$0	\$738,005

Distribution of Annuity Income at Age 65

10th Percentile	\$0	\$48,708	\$57,122
Median	\$0	\$48,708	\$57,122
90th Percentile	\$0	\$48,708	\$57,122

Distribution of Systematic Withdrawal Income at Age 65

10th Percentile	\$32,131	\$6,301	\$114
Median	\$58,556	\$32,726	\$24,912
90th Percentile	\$116,021	\$90,190	\$77,425

Distribution of Total Income at Age 65			% change from Scenario 1		% change from Scenario 1
10th Percentile	\$32,131	\$55,009	71%	\$57,236	78%
Median	\$58,556	\$81,434	39%	\$82,034	40%
90th Percentile	\$116,021	\$138,899	20%	\$134,547	16%

Distribution of Legacy Wealth at Age 66 (After Any Annuitization Takes Place)

10th Percentile	\$909,917	\$178,974	-80%	\$745,687	-18%
Median	\$1,688,788	\$940,551	-44%	\$1,462,842	-13%
90th Percentile	\$3,394,901	\$2,634,624	-22%	\$3,009,941	-11%

Distribution of Legacy Wealth at Age 100

10th Percentile	\$0	\$0	---	\$1,271,352	+++
Median	\$649,780	\$217,897	-66%	\$2,132,234	228%
90th Percentile	\$8,981,503	\$5,851,818	-35%	\$7,793,538	-13%

Table 2 outlines the retirement outcomes for Steve and Susie in pre-tax dollars. The first part of the table summarizes how they allocate their savings between insurance and Steve's 401(k) for the three scenarios, as described earlier. To better understand the impacts of investment volatility on the upside and downside, Monte Carlo simulations are used to create a distribution of outcomes. The table reports the 10th percentile, median, and 90th percentile from this distribution. We can interpret the 10th percentile outcome as a bad luck case with poor investment returns. It is possible that retirement outcomes could be even worse, but generally Steve and Susie could expect better retirement outcomes than seen at the 10th percentile. The median reflects more typical outcomes. It is the midpoint of the distribution, with a 50% chance for worse outcomes and a 50% chance for better outcomes. These are reasonable outcomes for Steve and Susie to expect. The 90th percentile is a good luck outcome in which investments perform very well, supporting greater spending and larger account balances. These numbers represent the upside potential for when investments perform very well, but Steve and Susie should understand that it is unlikely for them to see such great results for their investments.

Scenario 1 presents the strategy for buying term insurance and investing the difference in a target date fund. In after-tax terms at retirement, the wealth accumulation ranges from \$918k at the 10th percentile to \$3.31 million at the 90th percentile, with a median outcome of \$1.67 million.¹ With a 3.5% withdrawal rate, this translates into taxable income ranging from \$32,131 to \$116,021, with a median income of \$58,556. The table also shows the distribution of

¹ Results are presented in terms of nominal dollars to avoid reader confusion about why inflation-adjusted dollars are less than nominal dollars. This decision does not impact any comparisons for the relative outcomes between scenarios. However, readers should understand that the purchasing power of a given amount of income or wealth will be less in the future. For today's 35 year olds, the real purchasing power of money will be about half of what it is today at age 65, and about 20% of today at age 100.

investment assets, which is the same as legacy wealth when there is no term life insurance², one year later at age 66. This is less relevant in Scenario 1 since there is no annuitization or death benefit. What is relevant is the distribution of legacy wealth at age 100. At the 10th percentile, the portfolio was already depleted. Steve and Susie would have no income or assets from this part of their household balance sheet. At the median outcome, about \$650k remains in the portfolio. And if markets perform very well so that much upside investment growth can be enjoyed, legacy wealth is nearly \$9.0 million.

Scenario 2 also uses term life insurance, but the difference is that partial annuitization takes place with a joint life SPIA at the retirement date. At retirement, the distribution of investment assets is the same as in Scenario 1. A joint-life SPIA with a 6.6% payout rate is purchased with assets of \$738k in pre-tax equivalent terms. (The \$553k whole life death benefit is equal to \$738k in pre-tax equivalent terms using a 25% tax rate) As assets are at least this large across the distribution, annuity income is \$48,708 across the distribution as well, in pre-tax terms. A 3.5% withdrawal rate is then used with any remaining assets to generate additional retirement income for Steve and Susie. Scenario 2 supports total income at the retirement date ranging from \$55,009 to \$138,899 at the 10th and 90th percentiles, with a median income of \$81,434. These numbers are larger than in Scenario 1 (71% larger at the 10th percentile, 39% larger at the median, and 20% percent larger at the 90th percentile), showing the power of mortality credits through the income annuity relative to an investments-only strategy. Because partial annuitization with a life only SPIA removes \$738k in pre-tax equivalent terms from investments, the distribution of legacy wealth at age 66 is much less (80% to 22% less from the

² Legacy wealth consists of the after-tax value of any remaining financial assets in the investment portfolio and any life insurance death benefit.

10th to 90th percentile) than in Scenario 1. By age 100, the investment portfolio is also depleted at the 10th percentile, though annuity income is still available to the couple for as long as they live. Legacy wealth also remains less at the median and 90th percentiles, relative to Scenario 1. Scenario 2 has taken advantage of only one-type of “actuarial bond,” creating real trade-offs when comparing with Scenario 1. Scenario 2 can support more retirement income, but less legacy wealth.

Scenario 3 integrates investments with whole life insurance and income annuities. The beginning value of 401(k) assets at Steve and Susie’s retirement age 65 is less (19% less at the 10th percentile, 13% less at the median, and 11% percent less at the 90th percentile) than in Scenario 1 for two reasons. First, less was invested in the 401(k) so that some of the cash flow could be directed into the whole life insurance policy, which includes the cash value. The whole life insurance cash value is not counted as part of the 401(k) assets. Second, the asset allocation in the 401(k) is more aggressive because the cash value of life insurance is taken into consideration and treated as a bond when determining the 401(k) asset allocation. The cash value is \$257k at age 65 or \$343k when stated as a pre-tax equivalent, in Scenario 3. The cash value may provide additional flexibility which our analysis does not further consider.

We next consider how much income can be generated by these assets. With partial annuitization through a single-life SPIA with a 7.74% payout for an amount equal to the death benefit of the whole life policy at age 65, annuity income is \$57,122 across the distribution. The 3.5% withdrawal strategy is then applied to any remaining investment assets, generating additional income of \$114 to \$77,425 across this range of the distribution. Total retirement income at age 65 ranges from \$57,236 to \$134,547, with a median of \$82,034. Compared to

Scenario 1, retirement income is 78% larger at the 10th percentile, 40% larger at the median, and 16% larger at the 90th percentile.

Legacy wealth at age 66 accounts for the loss of assets through partial annuitization, but also for the death benefit from the whole life policy. These two factors mostly cancel one another. The situation for legacy wealth changes dramatically as retirement continues. By age 100, Scenario 3 still maintains a death benefit to support legacy, which means there is \$1,271,352 at the 10th percentile, compared to the \$0 we witnessed in Scenario 1 and 2. At the median, Scenario 3 supports legacy wealth of \$2,132,234 which is 228% larger than in Scenario 1. At the 90th percentile, investments have performed very well, and though legacy wealth is 13% less than Scenario 1, it is still nearly \$7.8 million. It is important to keep in mind, though, that Scenario 3 generated more income than Scenario 1 throughout retirement, and so these numbers are not directly comparable. It is only when investments do very well that the investment-only scenario can offer greater legacy wealth, though this can partly be attributed to the reduced income provided by the strategy.

Despite this important matter about the income differences, at age 100 there is still a 65% chance that the legacy wealth in Scenario 3 is larger than the legacy wealth in Scenario 1. This reflects the differences between the upside potential of investments and the integrated strategy of Scenario 3 which includes actuarial science. Perhaps surprisingly, there is only a 35% chance that investments perform well enough for legacy wealth to be larger in Scenario 1. And even in such cases, legacy wealth is still quite comfortably large in Scenario 3, and beneficiaries will likely still be pleased. We should really focus more on the other 65%, where investments have not performed well enough for legacy wealth to be larger in Scenario 1. These are cases when

legacy wealth will otherwise be smaller across the board, and so each additional dollar of legacy will have a bigger impact on lives of beneficiaries.

The integrated approach is actually able to provide more legacy wealth in these cases, while also supporting more retirement income. This is the meaning of greater efficiency. Readers may be surprised that it is not at all a clear-cut case that the upside growth potential of investments will be sufficient to beat a more integrated approach using actuarial science and mortality credits. Actually, the odds are against investments.

A final comment is in order about the comparisons between Scenario 2 and 3. Scenario 2 does use one type of actuarial bond in retirement: a joint-life SPIA. But it does not maintain life insurance into retirement. Behaviorally, Scenario 2 presents a difficult strategy for many retirees to accept. Despite potential improvements to their retirement income, retirees generally do not like to annuitize their assets in such a way. Meanwhile, Scenario 3 uses a single-life SPIA and maintains a death benefit with life insurance, which can essentially protect or “refund” the assets used for the annuity purchase. This combination should be more palatable for retirees. And when we compare Scenarios 2 and 3, we can observe that the available income is similar, while the legacy value is substantially different. The lack of a death benefit with Scenario 2 means that legacy assets are substantially less. Psychologically, many retirees will find it easier to contemplate adopting Scenario 3 over Scenario 2.

The Implications for 50-Year Olds

At 35, Steve and Susie were still far from retirement. How would these strategies work for James and Julie, a couple who is already 50 years old? We make the following modifications to answer this. James has \$625,000 in his 401(k) plan. James determines that increasing his life insurance benefit amount by an additional \$597,345 through his projected retirement at age 65

provides the additional protection James and Julie desire. He considers a 15-year term insurance policy with a level annual premium of \$1,311.50. James also considers a whole life policy through age 100, as illustrated by OneAmerica, with a potential for a premium offset utilizing non-guaranteed dividends at age 65. Premiums are \$13,500 annually. He has \$32,000 in today's dollars (which will adjust for inflation) to divide between his 401(k) and insurance policy over the subsequent 15 years. Otherwise, everything else is the same as with the previous case. We maintain the same projected SPIA rates at age 65 under the assumption that while there would be less time for interest rates to increase from their present level, there will also be fewer mortality improvements in this shorter period of time.

Table 3
Outcomes for the Three Scenarios -- 50 Year Old Case

Current 401(k) Savings at Age 50 \$625,000
Amount Set Aside for Savings and Insurance \$32,000

	Scenario 1 Investments No SPIA + Term Life	Scenario 2 Investments + Joint-Life SPIA + Term Life	Scenario 3 Investments + Single-Life SPIA + Whole Life
Term-Life Premiums	\$1,312	\$1,312	\$0
Whole-Life Premiums	\$0	\$0	\$13,500
Taxes Paid	\$437	\$437	\$4,500
Remaining Contribution to 401(k)	<u>\$30,251</u>	<u>\$30,251</u>	<u>\$14,000</u>
Total Annual Cash Flow	\$32,000	\$32,000	\$32,000

All Subsequent Values are Provided on a Pre-Tax Basis (Assuming a 25% tax rate)

Distribution of 401(k) Assets at Age 65			% change from Scenario 1		% change from Scenario 1
10th Percentile	\$1,193,852	\$1,193,852	0%	\$949,839	-20%
Median	\$1,860,172	\$1,860,172	0%	\$1,568,945	-16%
90th Percentile	\$2,956,199	\$2,956,199	0%	\$2,585,914	-13%

Illustrated Life Insurance Values at Age 65			
Cash Value	\$0	\$0	\$311,800
Death Benefit	\$0	\$0	\$937,784

Distribution of Annuity Income at Age 65			
10th Percentile	\$0	\$61,894	\$72,584
Median	\$0	\$61,894	\$72,584
90th Percentile	\$0	\$61,894	\$72,584

Distribution of Systematic Withdrawal Income at Age 65			
10th Percentile	\$41,785	\$8,962	\$422
Median	\$65,106	\$32,284	\$22,091
90th Percentile	\$103,467	\$70,645	\$57,685

Distribution of Total Income at Age 65			% change from Scenario 1		% change from Scenario 1
10th Percentile	\$41,785	\$70,856	70%	\$73,006	75%
Median	\$65,106	\$94,177	45%	\$94,675	45%
90th Percentile	\$103,467	\$132,538	28%	\$130,269	26%

Distribution of Legacy Wealth at Age 66 (After Any Annuitization Takes Place)					
10th Percentile	\$1,176,106	\$254,856	-78%	\$935,105	-20%
Median	\$1,866,302	\$924,120	-50%	\$1,561,538	-16%
90th Percentile	\$3,013,654	\$2,049,278	-32%	\$2,606,164	--14%

Distribution of Legacy Wealth at Age 100					
10th Percentile	\$0	\$0	---	\$909,143	+++
Median	\$265,164	\$65,981	-75%	\$1,460,439	451%
90th Percentile	\$6,908,747	\$3,753,047	-46%	\$5,167,639	-25%

Table 3 provides the basic details for James and Julie. We can observe the same trends as before. Scenario 3 has less remaining to invest in the 401(k), but its use of actuarial fixed income assets helps it to support more income than Scenario 1 across the distribution of outcomes. At the median, Scenario 3 actually offers 45% more income to the couple than Scenario 1, while the available legacy wealth at age 100 is 451% larger despite the higher income potential. Certainly 50 years old is not too late to start implementing these integrated planning techniques.

Conclusions

There is little in the way of broad agreement within the financial services profession about how to best approach the retirement income planning problem. Some favor investments while others favor insurance. In this paper, we have explored a more integrated approach which includes investments, income annuities, and whole life insurance. By strategically combining all three of these elements, the potential exists to develop more efficient retirement income strategies which are able to support a higher income level and a greater legacy wealth than investment-only solutions.

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- Research commissioned by OneAmerica®.

Appendix on Capital Market Expectations

The capital market expectations connect the historical averages from Robert Shiller's dataset together with the current market values for inflation and interest rates. This makes allowances for the fact that interest rates and inflation are currently far from their historical averages, but it also respects historical averages and does not force returns to remain low for the entire simulation.

Table A1 provides summary statistics for the historical data, which guides the Monte Carlo simulations for investment returns. A Cholesky decomposition is performed on a matrix of the normalized values for the risk premium, bond yields, home prices, bills and inflation. A Monte Carlo simulation is then used to create error terms for these variables, which preserve their contemporaneous correlations with one another. Then the variables are simulated with these errors using models that preserve key characteristics about serial correlation. Though home prices and bills are not used, I present the complete model which also takes them into account.

Table A1
Summary Statistics for U.S. Returns and Inflation Data, 1890-2013
Correlation Coefficients

	Arithmetic Means	Geometric Means	Standard Deviations	Stocks Returns	Risk Premium	Bond Yields	Bond Returns	Home Prices	Bills	Inflation
Stock Returns	10.7%	9.1%	18.3%	1	0.99	0.04	0.06	0.17	-0.09	0.06
Risk Premium	6.1%	4.4%	18.3%	0.99	1	-0.09	-0.01	0.15	-0.20	0.03
Bond Yields	4.7%	---	2.4%	0.04	-0.09	1	0.52	0.12	0.85	0.22
Bond Returns	4.9%	4.7%	6.7%	0.06	-0.01	0.52	1	-0.06	0.33	-0.09
Home Prices	3.3%	3.0%	7.4%	0.17	0.15	0.12	-0.06	1	0.03	0.37
Bills	4.5%	---	3.0%	-0.09	-0.20	0.85	0.33	0.03	1	0.14
Inflation	2.9%	2.8%	5.4%	0.06	0.03	0.22	-0.09	0.37	0.14	1

Source: Data from Robert Shiller's webpage. The U.S. S&P 500 index represents the stock market, 10-year Treasuries represent the bond index, the Shiller-Case home price index for homes, 6-month Treasuries for bills, and the Consumer Price Index for inflation.

With the correlated error terms, inflation is modeled as a first order autoregressive process starting from 1.58% inflation in 2013 and trending toward its historical average over time with

its historical volatility. Bond yields are similarly modeled with a first order autoregression with an initial seed value of 2.12%. Next, home prices and the risk premium are both modeled as random walks around their historical averages and with their historical volatilities. Bond returns are calculated from bond yields and changes in interest rates, assuming a bond mutual fund with equal holdings of past 10-year Treasury issues. Stock returns are calculated as the sum of bond yields and the equity premium over yields. As a final step to ease explanations in the text by simplifying the calculations of inflation-adjusted numbers, I replace the randomly generated inflation simulations with a fixed inflation rate equal to 2.5%. Figure A1 shows the medians for the key variables.

Figure A1
Medians of Simulated Outcomes for Inflation, Bonds, and Stocks

